Stimulated Emission of Radiation: an Example in Special Relativity for Undergraduate Years

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Masers and Lasers operate by stimulated emission, which is relativistic. Lasers are usually made of condensed matter, and are used in condensed matter research. Therefore stimulated emission itself is a proper topic for a condensed matter conference. At least three monographs on Special Relativity treat single atom absorption and (spontaneous) emission of a photon, but not stimulated emission. A treatment of this last is given. There are a number of possible difficulties, for even advanced undergraduates, that emerge: perhaps this was the reason for omitting the topic in these texts.

1. Introduction

The operating principle of Masers and Lasers is stimulated emission, which is relativistic. Lasers are now used extensively in many branches of condensed matter physics, and are usually made of condensed matter, so stimulated emission is properly a topic for a condensed matter conference, as is the teaching of it. In stimulated emission, an incoming photon stimulates an excited atom to emit another photon of identical frequency to the incoming one. The stimulated photon has the same frequency and direction as the incoming one, and the phase of the associated quantum fields (vector potentials) of both quanta are the same.

A chance conversation between the authors revealed the following: (1) at least 3 monographs on Special Relativity (SR) which treated single atom absorption and (spontaneous) emission of a photon did not treat stimulated emission (SE); (2) that none of the authors had seen SE treated by SR in their undergraduate courses; and (3) that they had omitted to treat SE by SR in their SR undergraduate courses to students. So a treatment was undertaken, and revealed a number of possible difficulties even for advanced undergraduates. This treatment is given below, and the difficulties discussed.

2. Stimulated emission treated by Special Relativity

For reasons that will become obvious during this treatment, we revisit *spontaneous emission* before treating stimulated emission. We work in one spatial dimension.

An atom of rest mass M_o in the laboratory frame emits a photon of momentum Q/c, where c is the speed of light. The atom then has energy M^*c^2, rest mass M_o^* and momentum M^*v, where v is the recoil velocity. So we have

Energy conservation $$M_oc^2 = M^*c^2 + Q \qquad (1a)$$

Momentum conservation $$0 = Q/c - M^*v \qquad (1b)$$

which can be solved for the unknowns. We do not yet discuss the solution.

For *stimulated emission*, we have an incoming photon of energy Q, and two outgoing photons. We can either postulate that the outgoing photons each have energy Q also, and check the solution of the new equations to see if it exists, or invoke the indistinguishability of the photons as required by quantum mechanics, and in addition have the three energies equal. Either way, Eqs. 1a and 1b now become

$$M_o c^2 + Q = M^* c^2 + 2Q \qquad (2a)$$

$$Q/c = 2Q/c - M^* v \qquad (2b)$$

It is obvious that Eqs. 1a and 2a are equivalent, as are Eqs. 1b and 2b. Therefore, the solution of the equations for stimulated emission is that for spontaneous emission, with the three photons having identical energies and being indistinguishable. Thus the indistinguishability of the photons is compatible with Special Relativity, and the atomic recoil is also taken care of.

We should also recall that the *atomic* properties determine the necessary photon energy, and that the width of the energy levels is <u>infinitesimal</u> in this treatment.

The solution for Q in terms of the energy difference

$$Q_o = (M_o - M^*)c^2$$

is:

$$Q = Q_o\{ 1 - Q_o/(2M_o c^2)\},$$

which is the effect of the atomic recoil.

A full treatment of the spontaneous emission case together with a good discussion of the atomic recoil is given in French's text on Special Relativity [1].

3. Discussion.

One may object from a mathematical perspective that Eqs. 2a and 2b are trivially identical to Eqs.1a and 1b because the addition of the second photon's energy (respectively, momentum) to both sides of these equations amounts to no change at all in the said equations. This result is <u>non-trivial</u> from a physics perspective, however. While the second photon may be 'cancelled' from Eqns.2a and 2b in rendering them identical to Eqs. 1a and 1b, the 'history' of the 'uncancelled photon'is quite different. It is a *spontaneous emission photon* in Eqns. 1a/1b, and a *stimulated emission photon* in Eqs. 2a/2b. The fact that these two 'uncancelled' photons have identical energy and momentum serves to highlight the previously mentioned fact that the atom 'selects' the energy and momentum of the stimulated emission photon.

With the above points in mind, it is evident that no further calculation is required, in the analysis presented here, to pass from the SR treatment of spontaneous emission to the corresponding treatment of stimulated emission. We feel that these points regarding stimulated emission could be usefully and readily incorporated into intermediate-level undergraduate studies on Special Relativity.

References

[1] French A P 1968 *Special Relativity* (New York: Norton & Co Inc)